International Journal of Mechanical and Production Engineering Research and Development (IJMPERD)
ISSN (P): 2249-6890; ISSN (E): 2249-8001
Vol. 7, Issue 5, Oct 2017, 51-62

VIBRATION ANALYSIS OF COMPOSITE LAMINATED PLATES USING HIGHER ORDER THEORY AND LEVY'S SOLUTION

G. S. VIVEK[1], CH. LAKSHMI TULASI[2], L. PRAVEEN[3] & S. LOKESH[4]

[1,4]*Department of Mechanical Engineering, Institute of Aeronautical Engineering, Dundigal, Hyderabad, India*

[2]*Department of Mechanical Engineering, Chadalawada Ramanamma Engineering College, Tirupati, Andhra Pradesh, India*

[3]*Department of Mechanical Engineering, MLR Institute of Technology, Dundigal, Hyderabad, India*

ABSTRACT

This paper represents the exploration on the response of a symmetric composite laminated plate. The properties like weight reduction, Fatigue life, wear resistance, Corrosion resistance, Strength Stiffness and Thermal properties can be improved by forming the composite materials. Since vibration and composite material are two focal growing research topics, almost all the structural apparatus subjected to dynamic loading in their working life and vibration affects the working life of the structure. The most commonly used plate theories are Kirchhoff or classical laminated plate theory of thin plate, Mindlin-Reissene or First order shear deformation theory for moderate plates, Levinson's theory or Third order shear deformation theory. CLPT is the simplest plate theory, which the effect of transverse shear deformation that results in the underestimation of transverse deflection of the plate. FSDT is suitable for analyzing moderately thick plates. However, this theory does not satisfy the stress-free boundary conditions on the surface of the plate so it requires shear correction factors for the analysis of the thick plates TSDT is used. The higher order theory of composite laminated plates using levy solution is developed.

KEYWORDS: Laminated Composite Plate, Vibration Analysis, Levy's Solution & Higher Order Theory

Received: Jul 24, 2017; **Accepted**: Aug 12, 2017; **Published**: Aug 28, 2017; **Paper Id**.: IJMPERDOCT20175

Original Article

1. INTRODUCTION

Composite materials are the materials formed by combining two or more materials to achieve better engineering properties. The properties like weight reduction, Fatigue life, wear resistance, Corrosion resistance, Strength Stiffness and Thermal properties can be improved by forming the composite materials. The man made composite materials are from the two materials such as Reinforcement material called Fibres and the Base material called Matrix material. Matrix materials keep the fibres together which acts as load transfer medium between fibres. The Matrix's material have their usual bulk-form properties where as fibres have directionally dependent properties. The load transfer between the matrix material and fibre takes place through shear stress. Some of the application of composite materials are Automotive industry, Marine vessels, Machine elements and Energy application, Consumer durable products, Land transportation, Electrical electronics and communication.

2. ANALYTICAL SOLUTIONS OF COMPOSITE LAMINATED PLATE USING HIGHER ORDER THEORIES WITH LEVY SOLUTION

Composite laminates are constructed by stacking several unidirectional layers in a specific sequence of orientation. Hence the failure of the single layer does not give total failure of the laminate. However, it leads to progressive failure of the laminate. Failure mechanism of the composite materials is much more complicated than

that of the conventional materials due to heterogeneous construction. In case of isotropic materials the failure is observable on the surface of the specimen. In composite due to stacking of several layers failures are confined to one or few layers and may never penetrate to the surface of the laminate. The common mode of failure in composite materials is matrix cracking, fibres breakage and delaminating. A method of accurate analysis is essential for the design and analysis of such a composite laminated plates. Analytical solutions are developed for the first time for anti-symmetric cross ply and angle ply laminated plates based on the higher-order displacement for Mobel-1. This solution method used here is Levy Method with the state space approach, depending on the boundary condition. The levy solutions can be developed for the plate with two opposite edges with simply supported and the remaining two edges having any possible combinations of boundary condition i.e. free simply supported or fixed support. The numerical methods can be used to determine approximate solutions for more general boundary conditions.

3. LEVY SOLUTIONS FOR CROSS- PLY LAMINATED PLATES

The levy method can be used to solve the governing equation of a various plate theories for rectangular laminate in which two opposite parallel edges are simply supported and the other two edges can have any boundary condition. In the present work the levy type solution for Vibration of anti-symmetric cross- ply laminates are presented.

A rectangle laminate which has an even number of orthotropic layers is considered. The planar dimension is taken to be a, b and total thickness h. Here it is assumed that the edges y = 0, b is simply supported and the other edges can each have arbitrary boundary conditions. The laminate coordinate system (x, y, z) is taken such that $-a/2 \leq x \leq a/2$, $0 \leq y \leq b$, $-h/2 \leq z \leq h/2$.

The simply supported boundary condition (SS-1) on the edges y=0, b is expressed as:

$u_0 = w_0 = \theta_x = M_y = 0,\ u_0^* = w_0^* = \theta_x^* = M_y^* = 0$

The simply supported (S) clamped(C) and free (F) boundary conditions at the edges x = ±a/2 can be expressed as:

Simple supported S: $v_0 = w_o = \theta_y = N_x = M_x = 0,\ v_0^* = w_0^* = \theta_y^* = M_x^* = N_x^* = 0.$

Clamped C: $u_0 = v_0 = w_0 = \theta_x = \theta_y = 0,\ u_0^* = v_{0=}^* w_0^* = \theta_x^* = \theta_Y^* = 0$

Free F: $M_x = M_{xy} = N_x = N_{xy} = Q_x = 0,\ M_x^* = M_{xy=}^* N_x^* = N_{xy}^* = 0$

The generalized displacements expressed as products of undetermined functions and known trigonometric function which has to satisfy the simply supported the boundary conditions at y = 0, b.

The boundary conditions are satisfied by the following expression:

$$u_0\,(x, y, t) = \sum_{m=1}^{\infty} U_m(x,t) \quad \sin \beta y$$

$$v_0\,(x, y, t) = \sum_{m=1}^{\infty} V_m(x,t) \ \cos \beta y$$

$$w_0\,(x, y, t) = \sum_{m=1}^{\infty} W_m\,(x,t) \ \sin \beta y$$

$$\theta_x(x,y,t)=\sum_{m=1}^{\infty} X_m(x,t)\ \sin\beta y$$

$$\theta_y(x,y,t)=\sum_{m=1}^{\infty} Y_m(x,t)\ \cos\beta y$$

$$u_0^{\ *}(x,y,t)=\sum_{m=1}^{\infty} U_m^*(x,t)\ \sin\beta y$$

$$V_o^{\ *}(x,y,t)=\sum_{m=1}^{\infty} V_m^*(x,t)\ \cos\beta y$$

$$\theta_x^{\ *}(x,y,t)=\sum_{m=1}^{\infty} X_m^*(x,t)\ \sin\beta y$$

$$\theta_y^{\ *}(x,y,t)=\sum_{m=1}^{\infty} Y_m^*(x,t)\ \cos\beta y$$

The mechanical loads are also expanded in single Fourier sine series as:

$$q(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Q_m(x,t)\sin\beta y$$

Where $\beta = m\pi/b$

By substituting boundary conditions into the governing equation the differential equations are obtained as:

$$A_{11}U_{m,xx}+A_{14}U^*_{m,xx}+B_{11}X_{m,xx}+B_{14}X^*_{m,xx}=A_{33}\beta^2U_m+A_{36}\beta^2U^*_m+(A_{33}+A_{12})\beta V_{m,x}+(A_{15}+A_{36})\beta V^*_{m,x}+B_{33}\beta^2X_m+B_{36}\beta^2X^*_m+(B_{33}+B_{12})\beta Y_{m,x}+(B_{15}+B_{36})\beta Y^*_{m,x}-I_1\omega^2U_{m,xx}-I_2\omega^2X_{m,xx}-I_3\omega^2U^*_{m,xx}-I_4\omega^2X^*_{m,xx}$$

$$A_{33}V_{m,xx}+A_{36}V^*_{m,xx}+B_{33}Y_{m,xx}+B_{36}Y^*_{m,xx}=-(A_{33}+A_{21})\beta U_{m,xx}-(A_{24}+A_{36})\beta U^*_{m,x}+A_{22}\beta^2V_m+A_{25}\beta^2V^*_m-(B_{21}+B_{33})\beta X_{m,x}-(B_{24}+B_{36})X^*_{m,x}+B_{22}\beta^2Y_m+B_{25}\beta^2Y^*_m-I_1\omega^2V_{m,xx}-I_2\omega^2Y_{m,xx}-I_3\omega^2V^*_{m,xx}-I_4\omega^2Y^*_{m,xx}$$

$$D^s_{11}W_{m,xx}=D^s_{22}\beta^2W_m-D^s_{13}U^*_{m,x}+2D^s_{24}\beta V_m-D^s_{11}X_{m,x}-3D^s_{15}X^*_{m,x}+D^s_{22}\beta Y_m+3D^s_{26}\beta Y^*_m-Q-I_1\omega^2W_{m,xx}$$

$$B_{11}U_{m,xx}+B_{14}U^*_{m,xx}+D_{11}X_{m,xx}+D_{14}X^*_{m,xx}=B_{33}\beta^2U_m+(B_{36}\beta^2+2D^s_{13})U^*_m+(B_{33}+B_{12})\beta V_{m,x}+(B_{15}+B_{36})\beta V^*_{m,x}+D^s_{11}W_{m,x}+(D_{33}\beta^2+D^s_{11})X_m+(D_{36}\beta^2+3D^s_{15})X^*_m+(D_{33}+D_{12})\beta Y_{m,x}+(D_{15}+D_{36})\beta Y^*_{m,x}-I_2\omega^2U_{m,xx}-I_3\omega^2X_{m,xx}-I_4\omega^2U^*_{m,xx}-I_5\omega^2X^*_{m,xx}$$

$$\begin{aligned}B_{33}V_{m,x}+B_{36}V^*_{m,xx}+D_{33}Y_{m,xx}+D_{36}\\ &=-(B_{23}+B_{21})\beta U_{m,x}-(B_{24}+B_{36})\beta U^*_{m,x}+B_{22}\beta^2V_m+(B_{25}\beta^2+2D^s_{24}V^*_m+D^s_{22}\beta W_m\\ &-(D_{21}+D_{33})\beta X_{m,x}-(D_{24}+D_{36})X^*_{m,x}+\left(D_{22}\beta^2+D^s_{22}\right)Y_m+\left(D_{25}\beta^2+3D^s_{26}\right)Y^*_m-I_2\omega^2V_{m,xx}\\ &-I_2\omega^2Y_{m,xx}-I_3\omega^2V^*_{m,xx}-I_4\omega^2Y^*_{m,xx}\end{aligned}$$

$$A_{41}U_{m,xx}+A_{44}U^*_{m,xx}+B_{44}X_{m,xx}+B_{44}X^*_{m,xx}=A_{63}\beta^2U_m+(A_{66}\beta^2+4D^s_{33})U^*_m+(A_{42}+A_{63})\beta V_{m,x}+(A_{45}+A_{66})\beta V^*_{m,x}+2D^s_{31}W_{m,x}+(B_{36}\beta^2+2D^s_{31})X_m+(B_{66}\beta^2+6D^s_{35})X^*_m+(B_{42}+B_{63})\beta Y_{m,x}+(B_{45}+B_{66})\beta Y^*_{m,x}-I_3\omega^2U_{m,xx}-I_4\omega^2X_{m,xx}-I_5\omega^2U^*_{m,xx}-I_6\omega^2X^*_{m,xx}$$

$$A_{63}V_{m,xx}+A_{66}V^*_{m,xx}+B_{63}Y_{m,xx}+B_{66}Y^*_{m,xx}=-(A_{51}+A_{63})\beta U_{m,x}-(A_{54}+A_{66})\beta U^*_{m,x}+A_{52}\beta^2V_m+(A_{25}\beta^2+4D^s_{44})V^*_m+2D^s_{42}\beta W_m-(B_{51}+B_{63})\beta X_{m,x}-(B_{54}+B_{66})X^*_{m,x}+(B_{52}\beta^2+2D^s_{42})Y_m+(B_{25}\beta^2+6D^s_{46})Y^*_m-I_3\omega^2V_{m,xx}-I_4\omega^2X_{m,xx}-I_5\omega^2V^*_{m,xx}-I_6\omega^2X^*_{m,xx}$$

$$B_{14}U_{m,xx}+B_{44}U^*_{m,xx}+D_{41}X_{m,xx}+D_{44}X^*_{m,xx}=B_{36}\beta^2U_m+(B_{66}\beta^2+9D^s_{55})U^*_m+(B_{24}+B_{36})\beta V_{m,x}+(B_{54}+B_{66})\beta V^*_{m,x}+3D^s_{31}W_{m,x}+(D_{63}\beta^2+3D^s_{51})X_m+(D_{66}\beta^2+9D^s_{55})X^*_m+(D_{42}+D_{12})\beta Y_{m,x}+(D_{45}+D_{66})\beta Y^*_{m,x}-I_4\omega^2U_{m,xx}-I_5\omega^2X_{m,xx}-\omega^2U^*_{m,xx}-I_7\omega^2X^*_{m,xx}$$

$$B_{36}V_{m,xx}+B_{66}V^*_{m,xx}+D_{63}Y_{m,xx}+D_{66}Y^*_{m,xx}=-(B_{15}+B_{36})\beta U_{m,x}-(B_{45}+B_{66})\beta U^*_{m,x}+B_{25}\beta^2V_m+(B_{55}\beta^2+6D^s_{64})V^*_m+3D^s_{62}\beta W_m-(D_{51}+D_{63})\beta X_{m,x}-(D_{54}+D_{66})X^*_{m,x}+(D_{25}\beta^2+3D^s_{62})Y_m+(D_{55}\beta^2+9D^s_{66})Y^*_m-I_4\omega^2V_{m,xx}-I_4\omega^2X_{m,xx}-I_5\omega^2V^*_{m,xx}-I_7\omega^2X^*_{m,xx}$$

Levy's Solutions are Calculated by Rewriting the Equation in the Matrix form as

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} & S_{17} & S_{18} & S_{19} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} & S_{27} & S_{28} & S_{29} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} & S_{37} & S_{38} & S_{39} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} & S_{47} & S_{48} & S_{49} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} & S_{57} & S_{58} & S_{59} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} & S_{67} & S_{68} & S_{69} \\ S_{71} & S_{72} & S_{73} & S_{74} & S_{75} & S_{76} & S_{77} & S_{78} & S_{79} \\ S_{81} & S_{82} & S_{83} & S_{84} & S_{85} & S_{86} & S_{87} & S_{88} & S_{99} \\ S_{91} & S_{92} & S_{93} & S_{94} & S_{95} & S_{96} & S_{97} & S_{98} & S_{99} \end{bmatrix} \begin{Bmatrix} U_m \\ V_m \\ W_m \\ X_m \\ Y_m \\ U^*_m \\ V^*_m \\ X^*_m \\ Y^*_m \end{Bmatrix} +$$

$$\begin{bmatrix} m_{11} & 0 & 0 & m_{14} & 0 & m_{16} & 0 & m_{18} & 0 \\ 0 & m_{22} & 0 & 0 & m_{25} & 0 & m_{27} & 0 & m_{29} \\ 0 & 0 & m_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ m_{41} & 0 & 0 & m_{44} & 0 & m_{46} & 0 & m_{48} & 0 \\ 0 & m_{52} & 0 & 0 & m_{55} & 0 & m_{57} & 0 & m_{59} \\ m_{61} & 0 & 0 & m_{64} & 0 & m_{66} & 0 & m_{68} & 0 \\ 0 & m_{72} & 0 & 0 & m_{75} & 0 & m_{77} & 0 & m_{79} \\ m_{81} & 0 & 0 & m_{84} & 0 & m_{86} & 0 & m_{88} & 0 \\ 0 & m_{92} & 0 & 0 & m_{95} & 0 & m_{97} & 0 & m_{99} \end{bmatrix} \begin{Bmatrix} U_m \\ V_m \\ W_m \\ X_m \\ Y_m \\ U^*_m \\ V^*_m \\ X^*_m \\ Y^*_m \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ Q_m \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

4. FREE VIBRATION ANALYSIS OF SIMPLY SUPPORTED ANTI-SYMMETRIC CROSS-PLY LAMINATED PLATES

For free vibration set the mechanical loads to zero and assume periodic solution of the form:

$$U_{mn}(t)=U_{mn}e^{-i\omega t}$$

$V_{mn}(t) = V_{mn}e^{-i\omega t}$

$W_{mn}(t) = W_{mn}e^{-i\omega t}$

$X_{mn}(t) = X_{mn}e^{-i\omega t}$

$Y_{mn}(t) = Y_{mn}e^{-i\omega t}$

For free vibration, reduces to the eigen values as

$([S] - \omega^2[M])\{\Delta\}$

Where

$\Delta = U_{mn}, V_{mn}, W_{mn}, X_{mn}, Y_{mn}$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} & S_{17} & S_{18} & S_{19} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} & S_{27} & S_{28} & S_{29} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} & S_{37} & S_{38} & S_{39} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} & S_{47} & S_{48} & S_{49} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} & S_{57} & S_{58} & S_{59} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} & S_{67} & S_{68} & S_{69} \\ S_{71} & S_{72} & S_{73} & S_{74} & S_{75} & S_{76} & S_{77} & S_{78} & S_{79} \\ S_{81} & S_{82} & S_{83} & S_{84} & S_{85} & S_{86} & S_{87} & S_{88} & S_{99} \\ S_{91} & S_{92} & S_{93} & S_{94} & S_{95} & S_{96} & S_{97} & S_{98} & S_{99} \end{bmatrix} \begin{Bmatrix} U_m \\ V_m \\ W_m \\ X_m \\ Y_m \\ U_m^* \\ V_m^* \\ X_m^* \\ Y_m^* \end{Bmatrix} +$$

$$\begin{bmatrix} m_{11} & 0 & 0 & m_{14} & 0 & m_{16} & 0 & m_{18} & 0 \\ 0 & m_{22} & 0 & 0 & m_{25} & 0 & m_{27} & 0 & m_{29} \\ 0 & 0 & m_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ m_{41} & 0 & 0 & m_{44} & 0 & m_{46} & 0 & m_{48} & 0 \\ 0 & m_{52} & 0 & 0 & m_{55} & 0 & m_{57} & 0 & m_{59} \\ m_{61} & 0 & 0 & m_{64} & 0 & m_{66} & 0 & m_{68} & 0 \\ 0 & m_{72} & 0 & 0 & m_{75} & 0 & m_{77} & 0 & m_{79} \\ m_{81} & 0 & 0 & m_{84} & 0 & m_{86} & 0 & m_{88} & 0 \\ 0 & m_{92} & 0 & 0 & m_{95} & 0 & m_{97} & 0 & m_{99} \end{bmatrix} \begin{Bmatrix} U_m \\ V_m \\ W_m \\ X_m \\ Y_m \\ U_m^* \\ V_m^* \\ X_m^* \\ Y_m^* \end{Bmatrix}$$

For a non trivial solution, $\{\Delta\} \neq 0$, the determent of the coefficient matrix should be zero, which yields the characteristic equation:

$[S]-\lambda[M]=0$

Where $\lambda=\omega^2$ Is the Eigen value.

The real positive roots give the square of the natural frequency $\omega_{m,n}$ Associated with mode (m, n). The smallest of the equation is called the fundamental natural frequency.

5. RESULTS AND DISCUSSIONS

Numerical computations are carried out for the free undamped transverse vibration analysis of laminated

composite plate. The effect of material orthotropy, transverse shear deformation, the ratio of span to thickness, coupling between stretching and bending and the number of lamina in the laminate on the frequencies are investigated. The material properties used for each lamina of the laminated composite plate are as follows:

Elastic Layer (Graphite/Epoxy):

Modular ratio $E_1/E_2 = 3$ N/ cm^2

$G_{12} = G_{13} = 0.6E_2$ $G_{23} = 0.5E_2$

Young's Modulus $E_2 = E_3 = 10^6$ N/ cm^2

$\mu_{12} = \mu_{23} = \mu_{13} = 0.25$

The plane stress reduced elastic constant of the L^{th} laminates are:

C_{11} = 3.075 X 10^6

C_{12} = 0.0300 X 10^6

C_{22} = 0.9975 X 10^6

C_{33} = 0.6 X 10^6

C_{44} = 0.5 X 10^6

C_{55} = 0.6 X 10^6

The natural frequencies of general rectangular composites are presented here in non-dimensional form using the following multiplier $\varpi = (\omega a^2/h)\sqrt{\rho/E_2}$.

A simply supported laminated cross-ply composite plate of is considered for comparisons of fundamental natural frequencies. The set of orthotropic material properties by considering ${}^{E_1}/{}_{E_2}$ = open and the results are presented in Table 1. The numerical values of non-dimensioned fundamental frequencies obtained using the levy method for various boundary conditions are presented in the tables for anti-symmetric cross-ply laminated plates. The fundamental frequencies increase with increasing orthotropy ${}^{E_1}/{}_{E_2}$ as well as the number of layers.

The Effect of Orthotropy, Number of Layers, Side Thickness Ratio, Shear Deformation and Coupling on Fundamental Natural Frequency

- Figure 1 shows the effect of modulus ratio is more significant and there is a difference between 2, 4, 6 and 8 layered solution increases with moduli ratio. For an example the 4-layered laminated plate has a vibration frequency about 20% lower than those 6 and 8 layered plates with the same total thickness at a higher modulus ratio (E1/E2). It is also observed that with an increase in the number of layers the frequencies approach those of orthotropic plates.
- Figure 2 shows the effect of fundamental natural frequencies of the anti-symmetric cross-ply laminates is shown as a function of side to side thickness ratio and number layers are plotted and it is shown that the shear deformation effect is increasing significantly on the vibration of the plates. The effect of shear deformation

decreases with increasing values of a/h which is lesser than of layer 4,6 and 8

- Figure 3 shows the effect of non-dimensioned fundamental frequencies for the anti-symmetric cross-ply laminated plates as a function of the number of layers and side to thickness ratio (a/h). As the number of layers increases without changing the total thickness, thus increase the fundamental frequencies.
- Figure 4 shows the effect of thickness and the number of layers on the non-dimensioned fundamental frequencies for the layer 2, 4, 6, and 8 anti-symmetric cross-ply. It is also observed that the responses of the layered 2, 4, 6 and 8 laminate decreases in bending coupling and increases the fundamental frequencies.
- Figure 5 shows the effect of the plots of fundamental frequencies is a function of aspect ratio (a/b) and side to thickness ratio (a/h) for anti-symmetric cross-ply laminated composite plates. It is found that the percentage error is about 20% of the cross-ply laminated plates. This percentage error decreased with an increase in the number of layers.
- Figure 6 shows the effect of increasing the fundamental frequency. And it is noted that the fundamental frequency of the two layered plates is about 20% lower than that of the 8 layered laminate.

Table1: Non-Dimensionalized fundamental Frequencies $\omega = (\frac{\omega b^2}{h})\sqrt{\rho/E_2}$ for a simply Supported Anti-Symmetric Cross-ply Laminated Plate for a/h=5

No. of Layers	Sources	E_1/E_2				
		3	10	20	30	40
[0/90]2	Present	6.262	6.9597	7.7015	8.2670	8.7169
[0/90]4	Present	6.4952	8.1279	9.4480	10.2553	10.8058
[0/90]6	Present	6.5527	8.3666	9.8163	10.6954	11.2909
[0/90]8	Present	6.5824	8.4972	10.0257	10.9545	11.5859
[0/90]10	Present	6.6594	8.5688	10.5246	11.0269	11.8659

Table 2: Non-Dimensionalized fundamental Frequencies $\omega = (\frac{\omega b^2}{h})\sqrt{\rho/E_2}$ for a Simply Supported Anti-Symmetric Cross-Ply Square Laminated Plate for ${E_1}/{E_2}$=40

a/b	No. of layers	Source	a/h			
			10	20	30	40
1	0/90	Present	10.3923	11.3137	10.3871	11.3137
	(0/90) 2	Present	15.6205	17.4356	18.2884	18.5903
2	0/90	Present	25.1396	29.3938	30.5788	30.9838
	(0/90) 4	Present	34.1760	45.4313	49.0876	50.59654
3	0/90	Present	44.6766	58.2409	62.6105	64.4981
	(0/90) 6	Present	55.9285	84.6640	97.6241	104.0000
4	0/90	Present	66.2419	94.4880	105.7583	110.8513
	(0/90) 8	Present	78.7401	129.3677	157.7561	173.4359
5	0/90	Present	84.8065	148.9654	175.2316	185.2365
	[0/90]10	Present	98.3216	168.2546	195.5642	196.5469

Table 3: Variation of Non Dimensionalized Frequencies $\varpi = (\frac{\omega b^2}{h})\sqrt{\rho/E_2}$ with a/h for a Simply Supported Cross-Ply Square Laminated Plate ${E_1}/{E_2}$=40

No. of Layers	Source	2	4	10	20	50
(0/90)2	Present	5.0881	7.9091	10.412	11.0733	11.2999

(0/90)4	Present	5.404	9.2870	15.1058	17.6570	18.673
(0/90)6	Present	6.321	10.654	17.236	19.612	20.256
Table 3: Contd.,						
(0/90)8	Present	8.654	12.359	19.236	21.756	22.365
(0/90)10	Present	10.369	14.365	21.489	23.659	25.265

Table 4: Effect of Degree of Orthotropy of the Individual Layers on the Dimensionless Fundamental Frequenc of Simply Supported Anti Symmetric cross-Ply Square Laminated a/h=5 $\varpi = (\frac{\omega b^2}{h})\sqrt{\rho/E_2}$

No. of Layers	Sources	E_1/E_2				
		3	10	20	30	40
[0/90]2	Present	2.7483	2.7775	2.8082	2.8328	2.8533
[0/90]4	Present	2.7601	2.8289	2.8875	2.9247	2.9508
[0/90]6	Present	2.7622	2.836	2.8977	2.9353	2.9610
[0/90]8	Present	2.7633	2.8405	2.9025	2.9402	2.9657
[0/90]10	Present	2.7722	2.9302	2.9825	2.9524	2.9765

Table 5: Effect of Side-to-Thickness Ratio on the Non-Dimensional Frequencies $\omega = (\frac{\omega b^2}{h})\sqrt{\rho/E_2}$ of Anti-Symmteric Cross-Ply Square Plates

No. of Layers	a/h	Source	FS	FC	SS	SC	CC
2	5	Present	6.387	6.836	9.087	10.393	11.89
	10	Present	7.277	7.810	10.568	12.87	15.709
4	5	Present	8.288	8.966	11.673	12.514	13.568
	10	Present	11.074	11.863	15.771	18.175	20.831
6	5	Present	13.126	13.999	17.9771	20.359	22.936
	10	present	15.259	15.990	19.269	22.987	25.369
8	5	Present	16.269	17.489	21.874	23.658	29.269
	10	Present	18.329	19.359	23.148	25.489	30.569
10	5	Present	19.265	21.369	26.159	27.369	32.159
	10	Present	21.367	23.698	28.364	29.148	34.197

Table 6 Effect of the Orthotropic Ratio and Number of Layers on the Non-Dimensional Fundamental Frequency of Anti-Symmetric Cross-Ply Square Laminated at a/h=10 for Various Boundary Condition

No. Of layer	E1/E2	Source	FS	FC	SS	SC	CC
8	2	Present	7.037	10.746	12.245	6.310	7.374
	10	Present	8.11	12.368	15.395	7.589	8.521
	20	Present	8.954	13.689	15.99	8.525	9.410
	30	Present	9.647	13.761	117.216	9.274	10.139
	40	Present	10.249	15.674	18.218	9.951	10.772
10	2	Present	8.157	11.959	13.517	7.452	8.5
	10	Present	10.692	15.917	18.350	10.308	11.177
	20	Present	12.634	18.762	21.354	12.364	13.236
	30	Present	14.012	20.653	24.188	13.78	17.711
	40	Present	15.077	22.043	26.471	14.896	16.858

Table 7 Effect of Side-to-Thickness Ratio on the Dimensionless Frequencies, $\varpi = (\frac{\omega b^2}{h})\sqrt{\rho/E_2}$ of Anti symmetric Cross-Ply Square Plates

No. of Layer	a/h	Source	FS	FC	SS	SC	CC
8	5	Present	6.387	6.836	9.087	10.393	11.89
	10	Present	7.277	7.810	10.568	12.87	15.709
	5	Present	8.288	8.966	11.673	12.514	13.568

10	10	Present	11.074	11.863	15.771	18.175	20.831

Table 8: Non Dimensionalized Centre Deflection (ϖ) of Ant Symmetric Cross-Ply Square Plates with Various Boundary Conditions

No. of Layers	a/h	Source	FS	FC	SS	SC	CC
8	5	Present	2.213	1.739	1.668	1.337	1.090
	10	Present	1.660	1.189	1.219	0.850	0.619
10	5	Present	1.458	1.219	1.134	1.009	0.883
	10	Present	0.809	0.609	0.619	0.479	0.379

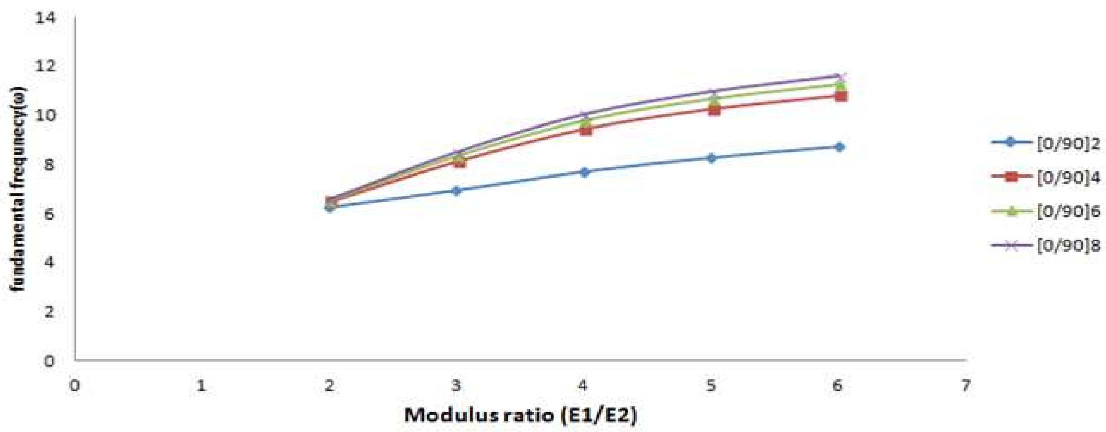

Figure 1: Non-Dimensional Fundamental Frequency (ω) Vs Modulus Ration (E_1/ E_2) for Simply Supported Anti-Symmetric Cross-Play Laminated Square Plate

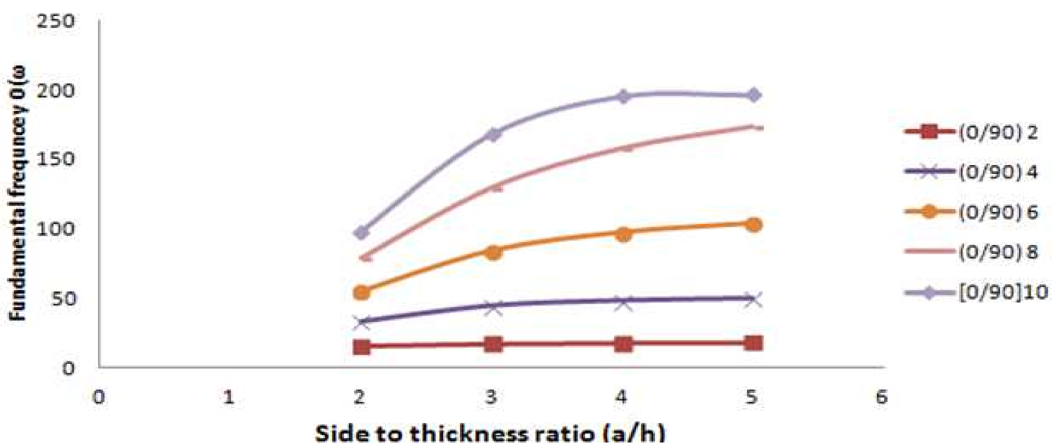

Figure 2: Non-Dimensionalized Fundamental Frequency (Ω) Vs Side to Thickness Ratio (A/H) for Simply Supported Anti-Symmetric Cross-Play Laminated Square Plate

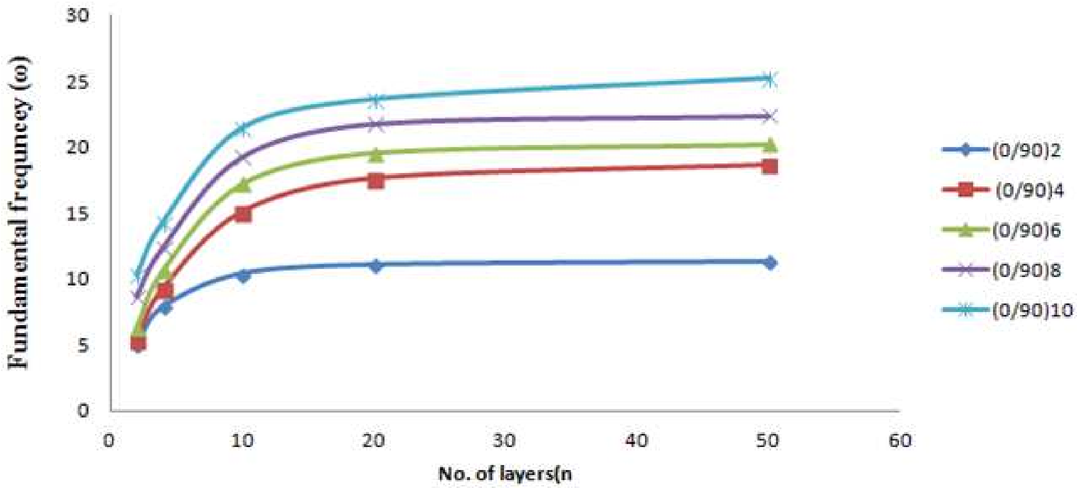

Figure 3: Non-Dimensionalized Fundamental Frequency (Ω) Vs Number of Layers (N) for Simply Supported Anti-Symmetric Cross-Play Laminated Square Plate

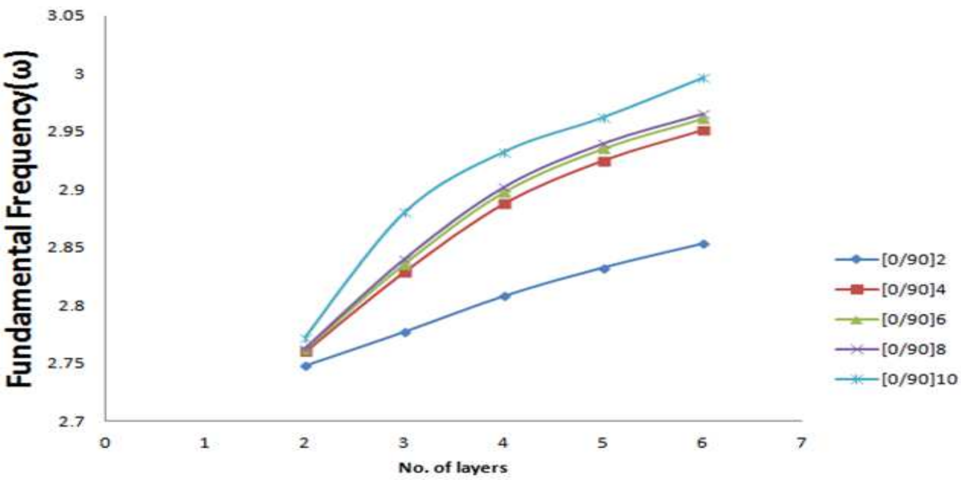

Figure 4: Non-Dimensionalized Fundamental Frequency (Ω) Vs Plate Aspect Ratio (A/B) for Simply Supported Anti-Symmetric Cross-Play Laminated Square Plate

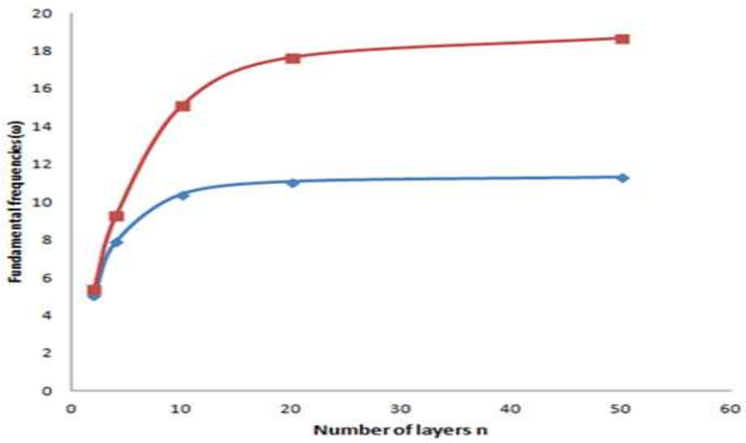

Figure 5: Non-Dimensionalized Fundamental Frequency (Ω) Vs Number of Layers (N) for Simply Supported Anti-Symmetric Cross-Play Laminated Square Plate

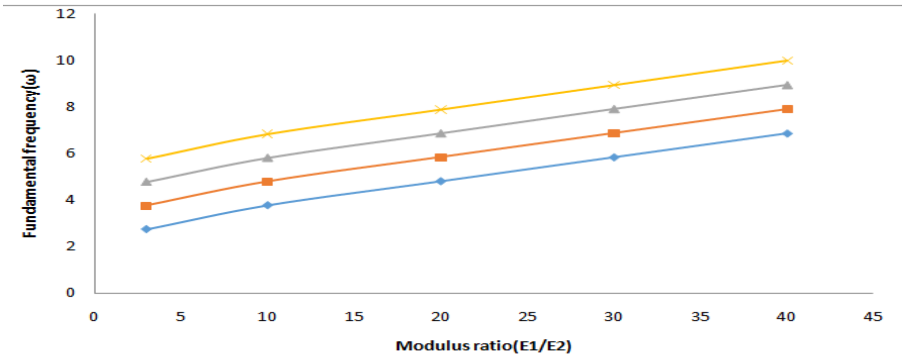

Figure 6: Non-Dimensionalized Fundamental Frequency (Ω) Vs Modulus Ratio (E_1/E_2) for Simply Supported Anti-Symmetric Cross-Play Laminated Square Plate

CONCLUSIONS

The following conclusions are drawn from the result of the investigations on composite anti-symmetric cross-ply laminated plate using higher-order theory with levy solution.

- The enhancement of thickness-length ratio increases the plate natural frequency for all mechanical and electrical boundary conditions.
- The close form of the natural frequency of an anti-symmetric cross-ply with two opposite edges and other two opposite edges having an arbitrarily boundary condition had been developed based on the levy solution.
- The higher order shear deformation theories with levy solution avoids slope discontinuities at the interfaces of the composite laminated plates and predicts more accurate transverse shear and normal deformation than the first order and other higher order theories considered and gives a much better approximation to the behaviour of laminated plates.
- Shear and normal deformation effects are very significant at the interface of the actuator and laminate, which cannot be ignored while modelling laminates, especially under mechanical loading.
- Non-linear variations of displacements, stresses and strains through the thickness of the laminate are experienced along with an approximation of transverse normal deformation and transverse shear deformations.
- Inter laminar stresses have been found to occur only within a local region near the geometric boundaries of a composite laminate which are frequently referred to as a free edge effect. The high inter laminar stresses coupled with the low inter laminar strength, have been found to be of critical significance in the failure of laminated composite structures due to delamination.

REFERENCES

1. *A. Bhimaraddi, L.K. Stevens, " A Higher order theory for Free Vibration of Orthotropic, Homogeneous and laminated Rectangular Plates," Of Applied Mechanics, Vol. 51, 1984*
2. *A.K. Noor, "Free vibration of multilayered composite plates," of composite materials, Vol, 7, 1939.*
3. *A.T. Jones, "Exact natural frequencies and model function for a thick off-axis lamina," of composite materials, Vol.5, 1971.*
4. *C.C. Lin, W.W. King, "Free transverse vibration of rectangular Unsymmetrical laminated plates," of sound and vibration, Vol. 36(1), 1974.*

5. *C.T. Sun, "Propagation of shock waves in Anisotropic composite plates," f composite materials, Vol.7, 1973.*
6. *C.V.J. Rao and A.K. Rao, "An exact analysis for vibration of simply supported homogeneous and laminated thick rectangular plates," of sound and vibration, Vol. 9, 1987.*
7. *C.W. Bert, T.L.C. Chen, "Effect f Shear Deformation on Vibration of anti-symmetric Angle-ply Laminated Rectangular Plates," Of solid structure, Vol. 14, 1978.*
8. *C.W. Bert, B.L. Mayberry, "Free vibration of unsymmetrical laminated anisotropic plates with clamped edges," of solid and structure.*
9. *C.Y. China, M. K. Prabhahara, "A general Mode approach to nonlinear flexural Vibration of Laminated Rectangular Plates." Of Applied Mechanics, Vol. 45, 1978.*
10. *E. Hinton, "A note on a thick strip method for the free vibration of laminated plates," of earth quake engineering and structural dynamics, Vol. 4, 1976.*
11. *J. A. Bennett, "Nonlinear vibration of simply supported cross-ply laminated plates," of composite materials, Vol.5, 1971.*
12. *J. A. Bennett, "Nonlinear vibration of simply supported cross-ply laminated plates," of Vol. 9, 1971*
13. *J.N. Reddy, "Free Vibration of Anti-symmetric, Cross-ply Laminated Plates Including Transverse Shear Deformation by the Finite Element Method." Of Sound and Vibration, Vol. 66(4), 1979.*
14. *J.N. Reddy, "A simple Higher-order Theory for Laminated Composite Plates," Of Applied Mechanics, Vol. 51, December 1984,*
15. *J.N. Reddy and N.D. Phan, "Stability and Vibration of Isotropic, Orthotropic and Laminated Plates according to a higher-order shear deformation theory," of sound and vibration, Vol. 98(2), 1985.*
16. *M. Di. Sciuva, "Vibration of Simply supported thick multilayered orthotropic plates: an Evaluation of a new displacement Model," Of Sound and Vibration, Vol. 105(1), 1986.*
17. *M. Levinson, "Free Vibration of Simply Supported Rectangular Plates: An Exact Elasticity Solution," of sound and vibration, Vol. 98(2), 1985.*
18. *N.D. Phan, J.N. Reddy, "Analysis of laminated Composite Plate using Higher-order shear Deformation Theory," Int. Of numerical Methods in Engineering, Vol. 21, 1985.*
19. *N.S. Putcha, J.N. Reddy, "Stability and Natural Vibration Analysis of Laminated Plates by using A mixed Element based on a refined plate theory," of sound and vibration, Vol. 104(2), 1986.*
20. *T. kuppusamy, J.N. Reddy, "Natural Vibration of Laminated anisotropic plates," Of sound and Vibration, Vol. 94(1), 1984.*
21. *R. Chandra, "Large Deflection Vibration of Cross-ply laminated plates with certain edges conditions," Of sound and vibration, Vol. 47(4), 1976.*
22. *R.G. Anderson, B.M. Irons and et al, "Vibration and Stability of plates using finite element," of solid and structure, Vol. 4, 1968.*
23. *S. Srinivas, A.K.Rao, "Vibration of simply supported thick orthotropic rectangular plates and laminates," of solids and structures, Vol.6, 1970.*